The solutions to problem (6), (7) are shown by dashed curves in Fig. 4 for $\delta = 3$ and Pe = 3.75 and 4.00. When the Prandtl number Pr $\to 0$, for all values of the parameters the critical Rayleigh number $Ra_* \to \infty$.

The results obtained here show that lateral injection of a reactant is an effective means of influencing the convective stability of reactive systems.

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NUMERICAL STUDY OF THE UNSTABLE INTERACTION OF A SUPERSONIC STREAM WITH A FLAT BARRIER

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UDC 532.525.2

Numerous experimental studies have been devoted to the interaction of an axisymmetric supersonic stream and a flat obstacle. For example, in [1-8], determinations were made of the boundaries of the zone of instability [7] and the amplitude–frequency characteristics of the process, and features of the pattern of flow were described qualitatively within a fairly broad range of modes of interaction. In [9-13], different hypotheses were advanced on the mechanism of the appearance of oscillations. Also, within the framework of different models, analytical solutions were constructed and were used to determine frequency characteristics of the process or the lower boundary of the stability zone. In [14], a numerical study was made of one mode of nonstationary interaction between a supersonic stream and a barrier of finite dimensions.

The present work examines the unstable interaction of a supersonic stream with an infinite barrier. The problem was solved within the framework of a model of a nonviscous, nonheat-conducting gas in accordance with the difference scheme of Godunov. The potential of this scheme for solving several problems of unsteady gasdynamics was examined in [15]. In [16], Godunov's method was successfully used to calculate stationary modes of interaction between a supersonic stream and a flat barrier.

The calculations were performed on a uniform rectangular grid. The distance from the symmetry axis to the top boundary of the grid N was made larger than the diameter of the maximum cross section of the first roll of the free stream determined from the data in [17].

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TABLE 1

Mode No.	M_a	n_a	$\varkappa_a$	k	Δ	x_b	r_b	m	c
I	1,5	8	1,4	1	0	9,0	∞	6	2400
II	2,0	3	1,4	1	0	6,5	∞	8	2000

Here, that part of the stream boundary which, according to [9], is a source of acoustic waves in the outer space is included in the calculated region. If the sound line in the stream flowing over the barrier went beyond the limits of the grid during the calculations, we increased the width of the grid accordingly.

The nozzle was simulated by a tube of thickness Δ with an open end. For the inside of the tube, we assigned constant values of the parameters of the gas, corresponding to Mach number M_a, off-design n_a, and index of the adiabatic curve $\varkappa_a$ of the flow at the edge of the shaped nozzle. We selected as the origin of the reactangular coordinate system rOx a point at the center of the outlet section of the tube. The calculated region of flow was bounded by the symmetry axis $r = 0$, the line $r = N$, and the sections $x = -k$ and $x = x_b$, where x_b is the distance from the nozzle edge to the barrier. Henceforth, all linear dimensions are noted relative to the nozzle radius r_a.

Parameters of the unperturbed atmosphere outside the nozzle are fixed at the initial moment of time. The boundary condition on the solid surfaces and symmetry axis was the condition of impermeability; on the free boundaries of the calculated field of values of the gas-dynamic parameters, we assumed that the values of the parameters in the cells adjacent to the boundary were equal. In principle, such an assignment of conditions at the boundaries of the calculated region does not maintain the original value of the off-design, and the off-design realized may differ from that assigned. This must be considered in analyzing the results obtained and comparing the calculated results with the empirical data. Close to the symmetry axis, we used an approximation of an equation for the radial component of velocity proposed in [18].

Table 1 shows the initial data for the calculated modes of unstable interaction with a barrier of radius $r_b = \infty$, as well as parameters of the calculating grid: m is the number of cells taken over the radius of the nozzle; c is the number of components of the grid.

During the calculations, at initial moments of time for each mode we observed transitional processes connected with the initial stage of interaction between the stream and barrier. Here, there was a nonmonotonic change in the parameters in the zone of interaction. The completion of the transitional processes was followed by the beginning of the mode of oscillations, nearly harmonic in character. Here, the realized off-design, defined as the ratio of the pressure at the nozzle edge to the mean (for the period) pressure at point $x = 0$, $r = N$, differed by no more than 5% from the initial value. There was subsequently some drift in this quantity toward smaller values.

The relative pressure pulsations $(p_{max} - p_{min})/p_{av}$, where p_{max}, p_{min}, and p_{av} are the maximum, minimum, and average (for the period) pressures at the point being examined in the calculated region, did not exceed 5% in the top left corner of the grid on modes I and II.

Mode I calculations were performed with a grid width $N = 6.5$ and 8.1. Increasing N did not produce any substantial changes in the qualitative pattern of flow in the region of interaction, at least in the first period after the "establishment" of oscillations. The values of pressure in the center of the barrier and the peripheral maximum of pressure at the barrier at $N = 8.1$ differed by less than 10% from the corresponding values at $N = 6.5$.

Table 2 shows theoretical and experimental values of certain amplitude—frequency characteristics of the pulsation process: Strouhal number $Sh = fr_a/a_0$ (f is the frequency of the process, a_0 is the speed of sound in unperturbed atmosphere), mean distance from the barrier to a central shock wave $\langle\varepsilon\rangle$, the doubled amplitude of the oscillations of the central shock wave $\Delta\varepsilon$, and the relative pressure pulsations in the center of the varrier $\Delta p/p_c$ (Δp and p_c are the variable and constant components of the pressure at the barrier, respectively). The subscript t denotes calculated values of the parameters of the pulsation process, while subscript e denotes experimental values. Data for values of the number Sh_e were taken from

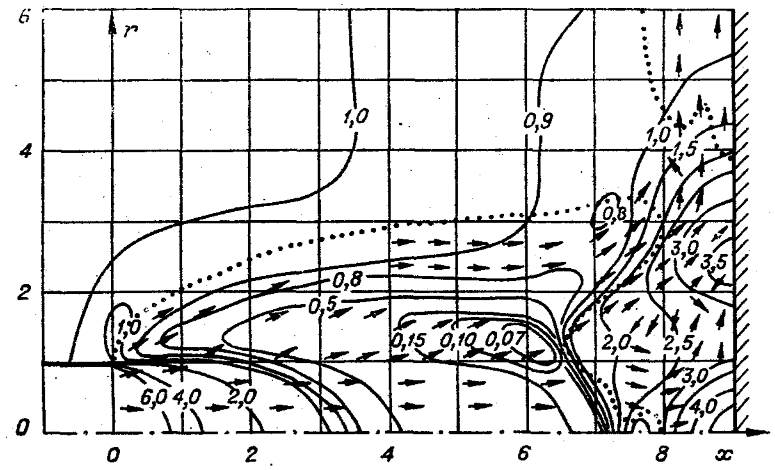

Fig. 1

TABLE 2

Mode No.	Sh_t	Sh_e	$\langle\varepsilon\rangle_t$	$\langle\varepsilon\rangle_e$	$\Delta\varepsilon_t$	$\Delta\varepsilon_e$	$\left(\frac{\Delta p}{p_c}\right)_t$	$\left(\frac{\Delta p}{p_c}\right)_e$
I	0,067	0,066	2,9	3,3	0,70	0,72	0,26	0,25
II	0,080	0,090	1,6	2,1	0,40	0,80	0,15	—

dependences cited in [4] (which was an approximation of the empirical data in [6]); the values of $\langle\varepsilon\rangle_e$ and $\Delta\varepsilon_e$ were obtained from Eqs. (16) and (17) in [6]. For mode II, the values of $\Delta\varepsilon_e$ and $(\Delta p/p_c)_e$ correspond to the empirical data presented in [8].

It is evident from Table 2 that the theoretical results agree satisfactorily with the empirical data with respect to frequency. The value of $\langle\varepsilon\rangle_t$ is lower than the corresponding quantity $\langle\varepsilon\rangle_e$. In mode II, the difference between $\langle\varepsilon\rangle_t$ and $\langle\varepsilon\rangle_e$ reaches 25% of the value of the latter. This is connected first of all with an insufficiently accurate calculation of the parameters on the initial supersonic section of the stream, particularly the inaccurate calculation, by Godunov's method, of the parameters in the centralized rarefication wave at the edges of the nozzle (e.g., see [19]) and the severe blurring of the stream boundaries in the case of a through calculation. The calculated values of the oscillation amplitudes and the drop-off of the shock, however, satisfy the relation obtained experimentally in [6].

To analyze the qualitative pattern of flow in the stream in front of the barrier (beyond the reflected and central shock waves), lines of constant values of pressure (solid), sound lines (points), and the field of velocity vector directions (arrows) are shown in Figs. 1-5 for modes I and II in the region $0 \le r \le N$, $x_b/2 \le x \le x_b$. The numbers on the isobars denote values of pressure p relative to the atmospheric pressure. At $p \ge 1.0$, the lines p = const are taken through $\Delta p = 0.5$. The region of crowding of the isobars denotes the position of shock waves. For the general pattern of flow, Fig. 1 contains isobars and sound lines for the entire calculated field for mode I. Figures 1-4 correspond to four successive phases of the oscillatory period on mode I: Figs. 1, 3 — minimum and maximum withdrawal of shock waves from the barrier; Figs. 2, 4 — average withdrawal of shock wave from barrier with movement of the wave toward the nozzle and the barrier, respectively. Figure 5 shows the pattern of flow in the stream in front of the barrier for mode II at the moment corresponding to minimum withdrawal of the shock wave from the barrier. Such a pattern of flow behind the central shock wave is also characteristic for mode II at the remaining moments of time. Figure 6a, b shows values of pressure in the center of the barrier p_0 (dashed line), the peripheral maximum of pressure at the barrier p_m (dash—dot line), and the distance from the barrier to the central shock wave ε (solid line) in relation to dimensionless time $\langle t\rangle = ta_0/r_a$ for modes I and II.

Analysis of the data obtained here shows that, in accordance with the well-known findings in [1, 3, 7], both mode I and mode II are characterized by the following: the equality of the frequencies of oscillation of the central shock wave and the pressure at the barrier; the presence of a peripheral pressure maximum at the barrier; the existence at different moments of time in the central region — bounded by the central shock wave, symmetry axis, barrier

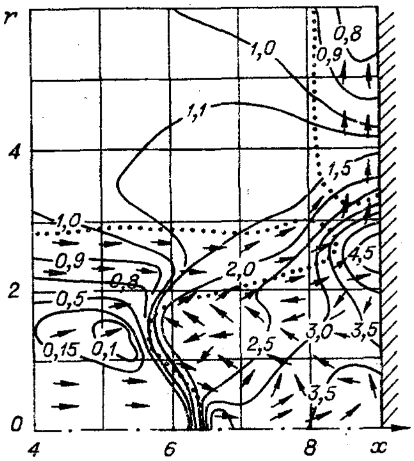

Fig. 2

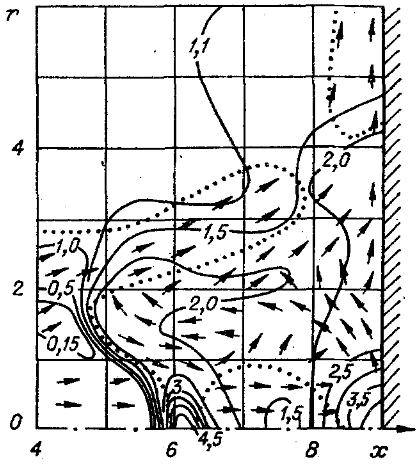

Fig. 3

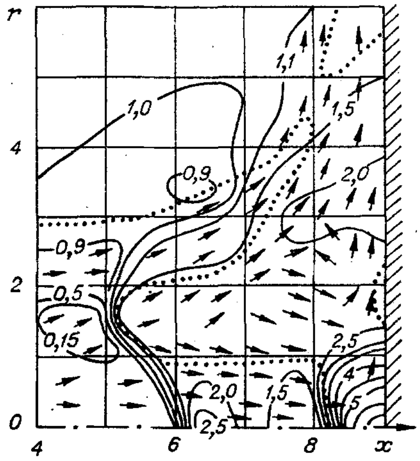

Fig. 4

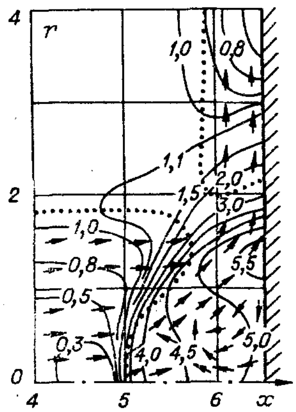

Fig. 5

surface, and the surface of tangential discontinuity — of a "return" flow of gas from the periphery to the center and a counterflow of gas from the barrier toward the nozzle; the gas flow beyond the reflected shock wave remains supersonic for the entire oscillatory period. The results of the calculations also show that there is a region of subsonic flow in the peripheral gas stream close to the barrier (Figs. 1-5).

There are also differences in the qualitative pattern of flow for modes I and II. In mode II, the return flow of gas from the periphery to the center reaches the symmetry axis, while the counterflow of gas from the barrier to the nozzle travels along the symmetry axis; in mode I, the return flow does not reach the axis, and the counterflow is circular. Here, the values of the peripheral maximum of pressure in mode II are always larger — and in mode I, always smaller — than the pressure in the center of the barrier (Fig. 6a and b).

Another feature of the pattern of flow on mode I compared to mode II is the existence of local supersonic zones in the central region at different moments. With movement of the central shock from the barrier to the nozzle, at different moments a supersonic zone appears in the subsonic flow of gas that passed through the central shock. This subsonic zone is localized near the axis of the stream. Its dimensions grow rapidly in proportion to the departure of the central shock from the barrier, and at the moment the shock begins to move toward the barrier this zone (subsonic) joins up with the supersonic zone formed behind the central shock (Figs. 3 and 4). Here, behind the central shock in the vicinity of the triple point, gas velocity remains subsonic. The presence of supersonic flow in the central region leads to the appearance of a second shock wave in front of the barrier (see Fig. 4).

The presence of a second shock wave in front of a barrier to an annular counterflow of gas under conditions similar to mode I was established empirically in [7]. The basic possibility of supersonic flow behind a central shock wave and the presence of a second shock

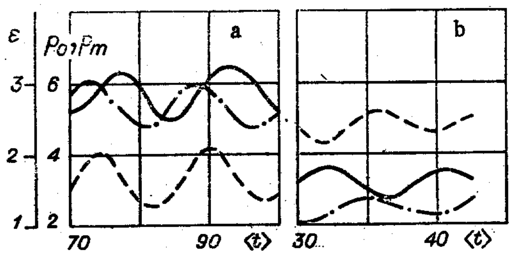

Fig. 6

wave in front of a barrier under conditions of unstable interaction were also noted in [10], although it was not mentioned that here the gas velocity behind the shock wave in the vicinity of the triple point remains subsonic for the entire period of oscillation.

In an analytical solution with a constant density in [20], it was shown that the empirically observed transition from a stationary mode of interaction to an unsteady mode corresponds to a change in the sign of the velocity component tangent to the central shock in the vicinity of the axis, the sign change being from + to – (where the positive direction is the direction from the axis). In the calculations, for both mode I and mode II this component is negative near the axis for the entire oscillatory period, but positive in the vicinity of the triple point. The point at which the central shock is orthogonal to the incoming flow coincides in the calculations with the point of inflection of the shock.

In the present work we examined conditions of severe instability far from the upper boundary of the instability zone (see [12]). According to [1, 3], viscosity may have a substantial effect on the parameters and pattern of flow in the zone of interaction near this boundary.

Numerical estimates of stationary modes of interaction made in [16] within the framework of a model of a nonviscous, non-heat-conduting gas showed good agreement with empirical data. Unsteady flow was not observed in [16] in a mode similar to conditions of severe instability (in the process of establishment of the mode, parameters throughout the region approached steady values); this could possibly be attributed to specific features of the calculation scheme, namely the fact that one of the boundaries of the moveable calculating grid coincided with the boundary of the stream. Thus, the effect of perturbations propagating in the external field on stream parameters was not considered.

The completed calculations provide evidence of the feasibility of the numerical investigation of unstable modes of interaction by the finite-difference method of Godunov with the inclusion of a nearby external field of the stream in the calculated region. The calculations allowed us to obtain detailed information on the pattern of flow and confirm and refine several conclusions reached earlier by experimental and theoretical methods.

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MODELING THE TURBULENT TRANSPORT OF AN IMPULSE IN THE WAKE OF A CYLINDER WITH THE USE OF EQUATIONS FOR THIRD MOMENTS

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1. One trend in modern phenomenological theory of turbulent transport is the formulation of a system of equations for the moments of the hydrodynamic fields of a turbulent flow, the maximum order of which is usually predicted with the aid of both physical considerations and the chosen method of closing the system. Models of turbulent transport have recently been proposed that are closed at the level of second moments — in which the unknowns are second moments — and in which third moments are modeled on the basis of heuristic considerations. Equations for moments of higher order are ultimately attractive for the reason that, in a whole range of physical problems, the turbulent transport of impulses, heat, or scalar properties cannot be correctly described within the framework of the simplest first-order gradient models (such as the Prandtl theory of displacement paths). Such problems are not the exception, and several of them may be found in [1-4]. An example of a model of turbulent transport closed at the level of the second moments (second-order model) would be the model [5] in which the turbulent flows (i.e., the second moments of turbulent fluctuations) are closed by means of the use of the method of the kinetic theory of gases in connection with third moments. Here, in essence a rough analogy is being made with kinetic theory, with the following justification: if a rough approximation for second-order moments makes it possible to compute first-order values in the simplest cases (this is true with the simplest phenomenological models of turbulent transport, based on the length of displacement paths), then it is possible that similar coarse approximations will make it possible to correctly predict second

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